Problem 2

For the following series, write formulas for the sequences a_n, S_n, and R_n, and find the limits of the sequences as $n \to \infty$ (if the limits exist).

$$\sum_0^\infty \frac{1}{5^n}$$

Solution

$$a_n = \frac{1}{5^n}$$

$$S_n = \sum_{i=0}^{n} \frac{1}{5^i} = \sum_{i=0}^{n} \left(\frac{1}{5}\right)^i = \frac{1 - \left(\frac{1}{5}\right)^{n+1}}{1 - \left(\frac{1}{5}\right)} = \frac{5}{4} - \frac{5^{-n}}{4}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(\frac{5}{4} - \frac{5^{-n}}{4}\right) = \frac{5}{4}$$

$$R_n = S - S_n = \frac{5}{4} - \left(\frac{5}{4} - \frac{5^{-n}}{4}\right) = \frac{5^{-n}}{4}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{5^n} = 0$$

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{5^{-n}}{4} = 0$$